

To Enjoy the Morning Flower in the Evening - What does the Appearance of Infinity in Physics Imply? *

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Abstract

A new regularization - renormalization method with no explicit divergence , no counterterm , no bare parameter and no arbitrary running mass scale is discussed . There is no difficulty of triviality and the Higgs mass in the standard model is calculated to be 138Gev .

1 Introduction

Beginning from the 1930s , the Quantum Mechanics (QM) evolved into the Quantum Field Theory (QFT) . Not only the electromagnetic field is quantized into the photon , but also the electron is further quantized in the second quantization . Both of them might be created or annihilated in mutual interaction process . Feynman innovated a diagram method to describe such kind of process , where some closed loop may occur in the diagram . In the covariant form of QFT , the particle on the loop is “virtual” in the sense of the relation among its mass , energy and momentum in free motion

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being violated. It is said that the particle is “off mass-shell”. The internal momentum of virtual particle should be integrated in the calculation, then one encountered the problem of divergence, i.e., the infinity, in physics.

For example, in the calculation of “self-energy” of electron in Quantum ElectroDynamics (QED), one considered a freely moving electron with (four dimensional) momentum p . The electron emits a virtual photon with momentum k and then absorbs it. This process may induce a modification of electron mass: $m \rightarrow m + \delta m$, with δm being the “radiative correction” of mass. The momentum is conserved at the two vertices. Thus the Feynman Diagram Integral (FDI) reads, ($e < 0, \hbar = c = 1$):

$$-i\Sigma(p) = -e^2 \int_0^1 dx [-2(1-x)\gamma_\mu p^\mu + 4m] I \quad (1)$$

$$I = \int \frac{d^4 K}{(2\pi)^4} \frac{1}{(K^2 - M^2)^2} \quad (2)$$

$$M^2 = p^2 x^2 + (m^2 - p^2)x \quad (3)$$

where x is the Feynman parameter, $K = k - xp$.

We shall concentrate on the integral I with the integration range of K being $(-\infty \rightarrow \infty)$. Since the denominator $(K^2 - M^2)^2 \sim K^4$, so the integral diverges as

$$I \sim \int \frac{dK^4}{K^4} \sim \int_0^\Lambda \frac{dK}{K} \quad (4)$$

where a radius Λ of large sphere in four dimensional space is introduced and is called as the cutoff of momentum in integration. When $\Lambda \rightarrow \infty$, Eq.(4) shows the logarithmic divergence of most weak type. In other FDIs, one may encounter the linear divergence ($\sim \Lambda$) and the quadratic divergence ($\sim \Lambda^2$) etc.

2 How to deal with the divergence?

The divergence can not correspond to the observable quantity in physics, so physicists devised a series of tricks to treat it. Let us look at the self-energy calculation of electron as an example again.

First , one needs to control the divergence , i.e.,to perform the regularization . The introduction of a cutoff Λ into the divergent integral may be the most naive one , but not satisfied due to its violation of Lorentz covariance in the theory . In recent 20 years , a most popular method for regularization is the so-called dimensional regularization scheme , where the dimension of space-time equals to $(4-\varepsilon)$ with ε being a continuously varying number approaching to positive zero . Hence the integral I can be formally expressed analytically by the Gamma function and the divergence is concentrating into the pole of Gamma function , i.e., into a term like $1/\varepsilon$.

Next , for cancelling the divergent term $1/\varepsilon$,one introduces a counterterm.

As a third step , because only the finite parameter (e.g. , the mass m) can be observed in the experiment whereas now a divergent counterterm appears , one has to introduce an another divergent bare parameter (e.g. , the bare mass m_0). The latter is combined with the counterterm to cancel the divergence and corresponds to the observed mass m_R ,which achieves the renormalization procedure .

An another interesting point is as follows . As the dimension of space-time has been continued analytically from 4 to $(4 - \varepsilon)$,the originally dimensionless (in natural unit system $\hbar = c = 1$) charge square e^2 acquires dimension . For making it dimensionless again , one manages to introduce a factor μ^ε with μ being an arbitrary parameter with mass dimension . After performing the renormalization procedure , the renormalized mass m_R or charge e_R will very with the running of mass scale μ . The choice of μ is ascribed to the choice of renormalization point , which corresponds to giving the observed physical parameter (m or e) a definite explanation .

3 The most difficult aspect in learning is inquiry

During the teaching and research process for over 40 years , I myself and many students raised the following questions again and again . How can we understand the physical meaning of changing the dimension of space-time from 4 to $(4 - \varepsilon)$? Is the mass generated from its "seed", the bare mass? Is the charge e evolving from the bare charge e_0 via the screening effect of vacuum polarization ? Why a new arbitrary mass scale μ would

emerge abruptly from an originally definite theory after quantization and renormalization ? Furthermore , when encountering $(1 + B)$ in calculation , one often replaced it by $(1 - B)^{-1}$. It is surely reasonable if B is much less than 1 . But now B is approaching infinity , is this reasonable ? Moreover , While e^2 remains finite , the bare one , e_0^2 , is infinite , but sometimes we substitute one by another when doing calculation at higher loop modification. Is this reasonable ? All these questions bothered me deeply .

From 1991-1994 , based on a lot of literatures [H .Epstein and V . Glaser, Ann . Inst . Henri Poincare 19 , 211 (1973) ; J . Collins , Renormalization , Cambridge University Press . (1984) ; J . Glimm and A . Jaffe , Collective Papers , Vol . 2 (1985) ; G . Scharf , Finite Electrodynamics , Springer-Verlag, Berlin , 1989 ; J . Dutch , F.Krahe and G . Scharf , Phys. Lett B 258, 457 (1991) ; D .Z.Freedmann , K .Johnson and J . I . Lattore , Nucl. Phys . B 371, 353 (1992)] , a PhD candidate , Ji-feng Yang proposed a new regularization-renormalization method [1] . It was rather simple and effective , substituting the infinity by some arbitrary constants . It was a break through , the situation was suddenly enlightened , Further , we have been studying it in applications and achieving a series of new cognition and results . Let me try to combine them together in the following for discussing with our readers .

In August 1997 , when I attended an International Conference at Nanjing University in memory of Professor C . S. Wu , I read the lecture delivered by Prof . Wu when she visited the Padova University , Italy , in 1984[2] .I was greatly inspired . She quoted Calileo's saying that a scientist should go beyond the "mere think " and must raise the "intelligent questions " via experiments . Yes , I had been merely thinking for a long time that "How can we find a new regularization method to deal with the divergence ? How can we pick up some thing fixed and finite for explaining the experimental data after the separation of divergence ?" Yet I had neither thought about " Why the divergence emerges ?" via the experimental facts , nor raised even more acute question that "If the divergent integral does not give us infinity, What would it yield ?"

Now I know the answer of the last question . It yields " Nothing " first but " Arbitrary constants " afterwards . Because if it does yield something fixed and finite , it must be wrong . The reason is as follows.

Suppose that after the separation of divergence from some divergent integral I , a finite contribution survives and is capable of corresponding to a tiny

but definite mass modification δm . Then one can write down $m = m_0 + \delta m$. Since m_0 is nonobservable , one may perform the calculation of higher order self-energy FDI for many times and then set m_0 approaching to zero . Hence one might claim that the mass m is generated via the radiative corrections from self-energy FDI . That would be “something generated from nothing ” . Actually , mass m has a dimension , say , m equals to 3 grams . It cannot be understood to create a mass with 3 grams from nothing . Only with the existence of an another definite mass scale — the standard weight of 1 gram, could the 3 grams be understood . This comprises a statement of “ principle of relativity ” in epistemology . In our point of view , it is nothing but the phase transition of the environment of particle , (i.e. , the vacuum) could provide the second mass scale and thus renders the generation of particle mass m possible . In this case we have to perform the nonperturbative calculation with the loop number L in FDI adding up to infinity in QFT [3] . On the other hand , the self-energy FDI with L finite has nothing to do with the mass generation .

4 The subtle use of infinity

Then I suddenly realized that the emergence of infinity given by a divergent integral is essentially a warning . It warns us that we had been expecting too much . We had been sticking to calculate obstinately some thing which is essentially not calculable .

For a long time I paid no attention to this warning . In studying the $\lambda\Phi^4$ model , the basis of standard model in particle physics , following many physicists , I also introduced a large but fixed cutoff as the substitute of infinity . I forgot what my mathematics teacher taught me 46 years ago . At that time , my teacher taught me about “infinity ” as follows , “what is it ? You first raise a large but fixed number N . Then I say it being larger than N . Next you replace N by another N' (larger than N) , I say it being even larger than N' . And so on so forth , its limit is infinity ” . Therefore , “infinity ” is not a very large and fixed number . Rather , it is a symbol .

Yes , the physical meaning of infinity in QFT is nothing but some thing we can't calculate . It implies a kind of “infinity ” which goes beyond the present boundary of our knowledge .

In some sense , the use of infinity in QFT just like that of imaginary

number unit $\sqrt{-1}$ in QM [4] . They have the subtlety with different tunes rendered with equal skill .

5 New Regularization - renormalization Method

With the previous sections as preliminary , the following trick for handling the divergence will be quite simple and natural . Let us return back to Eq.(2)

Taking the partial derivative of divergent integral I with respect to the parameter M^2 with dimension of mass square , we get

$$\frac{\partial I}{\partial M^2} = 2 \int \frac{d^4 K}{(2\pi)^4} \frac{1}{(K^2 - M^2)^3} \quad (5)$$

As the denominator has a behavior of K^6 , the integral becomes convergent now :

$$\frac{\partial I}{\partial M^2} = \frac{-i}{(4\pi)^2} \frac{1}{M^2} \quad (6)$$

For going back to I , we integrate Eq.(6) with respect to M^2 , yielding

$$I = \frac{-i}{(4\pi)^2} (\ln M^2 + C_1) = \frac{-i}{(4\pi)^2} \ln \frac{M^2}{\mu_1^2} \quad (7)$$

Here , according to the theory of indefinite integration , an arbitrary constant C_1 appears . We rewrite $C_1 = -\ln \mu_1^2$ with μ_1 carrying a mass dimension so that the argument of logarithmic function being dimensionless.

By means of the “ chain approximation ” , we can derive a renormalized mass $m_R = m + \delta m$. When the freely moving particle is on the mass-shell , i.e. , $p^2 = m^2$, we have ($\alpha \equiv e^2/4\pi$):

$$\delta m = \frac{\alpha m}{4\pi} \left(5 - 3 \ln \frac{m^2}{\mu_1^2} \right) \quad (8)$$

The arbitrary constant μ_1 is fixed as follows. We want the parameter m in the Lagrangian of original theory being still explained as the observed mass m_R . As mentioned in previous section , due to the disability of perturbative

QFT to calculate the mass , the latter can only be fixed by experiment . Hence , the condition $\delta m = 0$ gives

$$\ln \frac{m^2}{\mu_1^2} = 5/3, \mu_1 = e^{-5/6}m \quad (9)$$

The condition $m_R = m$ does not imply that we gain nothing from the calculation of self - energy FDI . When the motion of particle deviates from the mass - shell , i.e. $p^2 \neq m$, the combination of self - energy formula with other FDI_s in QED is capable of telling us a lot of knowledge . For instance, we are able to calculate quickly the (qualitative) energy shift of $2S_{1/2}$ state upward with respect to $2P_{1/2}$ state in Hydrogen atom being 997 MHz , so called Lamb shift (with experimental result 1057.8 MHz) [5] . The latter should be viewed as some mass modification for an electron in binding state. To be precise , by means of perturbative QFT , we can evaluate the mass modification but never the mass generation . This is a cognition we should keep in mind before we set up to use our method .

Based on this cognition , we replace the divergence by arbitrary constant μ_1 , then the latter is fixed by the mass m measured in the experiment . In some sense , this regularization trick renders the renormalization very easy , only one step is needed before putting into the place . No ambiguous concept like the counterterm and/or bare parameter is needed any more .

Here , two remarks are important :

(a) . There was the trick of taking derivative for reducing the degree of divergence of some integral I in previous literatures (as mentioned in sect. 3) . Only the elementary calculus for a freshman is needed in such kind of trick , it is rather simple. However, the crucial point lies in the fact that we should act from the beginning , act before the counterterm is introduced, act until the bottom is reached . That is , to take derivative of integral I with respect to M^2 (or to a parameter σ added by hand , say $M^2 \rightarrow M^2 + \sigma$) enough times until it becomes convergent , then perform the same times of integration with respect to M^2 (or σ then setting $\sigma \rightarrow 0$ again) for going back to I . Now instead of divergence , we obtain some arbitrary constant C_i . Note that one divergence is now resolved into some constants to be fixed . Each C_i has its unique meaning and role which makes the situation much clearer and well under control [6].

(b) . Some reader may comment that “The previous methods like the dimensional regularization scheme can also yield the result in conformity

with the experiment . Though some counterterm and bare parameter are introduced in the intermediate step , they are not observable and don't matter much . I think your method being not new. "In our opinion , when discussing the basic problem in physics , generally speaking , the simpler method is the more hopeful one to be correct , whereas the tedious one often proves to be incorrect as shown by many historical facts . Although the counterterm and bare parameter disappear at the final , the arbitrary running mass scale μ survives .The latter is hard to explain . How can a theory become unfixed to some extent after quantization and renormalization while it was totally fixed at the classical level ?

6 Is the $\lambda\Phi^4$ model trivial?

In a text book on QFT , the simplest scalar field model , the $\lambda\Phi^4$ model , is often discussed first . It is defined at the classical level by the Lagrangian density :

$$\begin{aligned}\mathcal{L} &= \left(\frac{\partial\Phi}{\partial t}\right)^2 - (\nabla\Phi)^2 - V(\Phi) \\ V(\Phi) &= \frac{1}{2}m^2\Phi^2 + \frac{1}{4!}\lambda\Phi^4\end{aligned}\tag{10}$$

where m is the particle mass excited at the symmetric vacuum $\Phi = 0$ and λ is a dimensionless (in natural unit system) coupling constant .

However , the $\lambda\Phi^4$ model with wrong sign in mass term is more useful in the standard model of particle physics , where $V(\Phi)$ is modified into

$$V(\Phi) = -\frac{1}{2}\sigma\Phi^2 + \frac{1}{4!}\lambda\Phi^4\tag{11}$$

Hence the ground state vacuum with lowest energy will be shifted from $\Phi = 0$ to $\Phi_1 = (6\sigma/\lambda)^{\frac{1}{2}}$. It is called as the spontaneously symmetry breaking (SSB) . The particle excited at the symmetry broken vacuum state has a mass $m_\sigma = (2\sigma)^{\frac{1}{2}}$.

Once the model undergoes the quantization , a difficulty arises . It was found that when the cutoff Λ goes to infinity , a singularity would occur in the solution of renormalization group equation (RGE) , which implies that the renormalized coupling constant λ_R approaches to zero . In other

words , there would be no interaction among particles . So it was thought as a difficulty of so called “triviality ” and the way out of the difficulty was setting a large value for Λ corresponding to a high energy , say $10^{15} Gev$ approximately . The $\lambda\Phi^4$ model should be treated as a low energy effective theory .

Things seem not so fair . Is there a similar problem in QED ? Why people have seldom talked about its difficulty of triviality ? Maybe the conformity between QED and experiments (e.g. , the Lamb shift as mentioned in sect. 5) is too accurate to be doubted .

We have restudied the $\lambda\Phi^4$ model with SSB according to the present point of view and found that there is no any triviality difficulty at all . The crucial point lies in the fact that any model in field theory defined by the Lagrangian at classical level needs to be redefined once it is quantized . As a metaphor , with a plane ticket at hand , one has to reconfirm it by a phone call before his departure from the airport . Here , our method to “reconfirm ” is choosing the two values of arbitrary constants emerged after the integration of FDI such that the position of broken vacuum Φ_1 and the particle mass m_σ excited on it remain unchanged at perturbative QFT level of any order .

Thus suddenly we saw the light . The difference between the $\lambda\Phi^4$ model with and without SSB lies in the essence that the former provides two mass scales , Φ_1 and m_σ , whereas the latter provides only one , m . Meanwhile, the invariant meaning of constant λ in the Lagrangian is not the coupling constant , the latter will change after quantization . For example , at one loop ($L = 1$) calculation , it reads

$$\lambda_R = \lambda \left[1 + \frac{9\lambda}{(32\pi^2)} \right] \quad (12)$$

The invariant meaning of λ is nothing but the ratio of two mass scales:

$$\lambda = 3 (m_\sigma/\Phi_1)^2 \quad (13)$$

which remains unchanged irrespective of the order of L even when L approaches to infinity .

It is more interesting to see that when performing the nonperturbative calculation in QFT , corresponding to add up approximately the contributions of FDI with L approaching to infinity (e.g. , by RGE method) , we find a singularity μ_c , a critical value in energy , emerging in our theory . If

the energy $E > \mu_c$, the original stable vacuum state Φ_1 would collapse into the state $\Phi = 0$. It could be called as the "Symmetry restoration" and the original model with SSB becomes ineffective. Therefore, we share the same opinion that $\lambda\Phi^4$ model is a low energy effective theory. However, the limitation in energy is not stemming from a finite cutoff Λ introduced by hand for avoiding the so-called difficulty of triviality. Rather, it is inherited from the intrinsic property of the model itself. Through out the calculations, we let the cutoff Λ running to infinity while the renormalized coupling constant λ_R remains normal. For example, in Eq. (12) with $L = 1$, λ_R is neither zero nor infinite.

The fact that there is no singularity in calculation when the loop number L is finite whereas some singularity emerges when L approaches to infinity has a deep meaning. As a contrast in mathematics, the geometric series $S_n = 1 + r + r^2 + \dots + r^n$ is analytic and has no singularity (except the point at infinity). But if n approaches to infinity, $\lim_{n \rightarrow \infty} S_n = 1/(1 - r)$ does have a singularity (a pole) at $r = 1$.

In our point of view, a physical model with some singularity is normal and nontrivial. On the other hand, a model without any singularity must be wrong, meaningless or trivial. The famous Liouville's theorem in the function theory with complex variable claims that if a function has no any singularity on the whole closed complex variable plane, it must be a trivial constant. An important nontrivial theory in physics is the general relativity, its singularity is nothing but the black hole.

Either the divergence in FDI or the singularity in the model of physics reminds us again and again of the fact that the world is infinite whereas our knowledge remains finite. Any model can at most provide some local or unilateral description of nature and the emergence of infinity or singularity just reflects the limit or boundary of our cognition. A famous Chinese philosopher Zhuangzi (369 BC~286 BC) said that "While my life span remains finite, the knowledge will extend beyond any boundary." What he was pondering is just an eternal contradiction in the process for cognizing the world by humanbeing.

7 Can the Higgs mass be predicted?

The discovery of top quark in 1995 was a great triumph of the standard model in particle physics once again. The coincidence between the theory and experiment has been improved year after year . One even can claim that the main purpose of present experiment will concentrate on searching for the Higgs particle , which is the consequence of coupling between the $SU(2) \lambda \Phi^4$ model with SSB and the gauge fields .

Can the mass of Higgs particle , m_H , be predicted ? This proves to be a challenging problem for theorists . For many years , only the upper bound and /or lower bound on m_H could be estimated (see , e.g., M. Sher, Phys. Rep.(1998) 179 , 273) . We had also attacked this problem in 10 years ago and obtained the following result [7] :

$$76\text{Gev} < m_H < 170\text{Gev} \quad (14)$$

where the lower bound seemed surprisingly high at that time. So some authors did not believe in it . However , the lower bound given by experiment already exceeded 65Gev in several years ago and now is approaching 90Gev . We are unsatisfied with our previous result , Eq(14) , now . In Ref [7] , we fixed the cutoff Λ to a large value and calculated m_H by old method , which proved to be a strenuous work and not a beautiful one . Now the situation is totally different . Based on the new point of view and new method, we arrive at a predicted value [8 , 9]:

$$m_H = 138\text{Gev} \quad (15)$$

quite fluently and clearly .

This value is found with other available experimental data as input and is located within the range constrained by the present phenomenological analysis on experiments .

We believe that the precision of prediction on m_H could be improved in accompanying with the progress in accuracy of the relevant experiments . As the energy now accessible by the accelerators is not far from 138Gev , it would be not too far to reveal the mystery of nature — if the Higgs particle can really be found in experiment ?

8 Concluding remarks

After diving and floating in the QFT for over 40 years , I “swam” eventually to a place where I can relax for a while . Eventually I am getting rid of the four puzzles —the explicit divergence , the counterterm , the bare parameter and the arbitrary running mass scale — which had bothered me for so many years. I begin to understand that “only one step is needed before putting into the place ” for renormalization , which is actually nothing but a procedure to “reconfirm ” something . In the long river for humanbeing to cognize the world , “ the way seems extremely long ”, we shall continue “ to search for (the truth) while keeping up and down ” (quoted from the poem by the famous Chinese poet and statesman Qu Yuan , 340 BC—278 BC) . But “ where we are located roughly ” seems comparatively clear . I am delighted in my mood which leads to a poem :

I’m now enjoying “Four No ” ,
It leaves only one step to go .
How subtle the “renormalization ” is ,
Just like to “reconfirm ” or so.

[Note] “Four No” means “no explicit divergence, no counterterm, no bare parameter and no arbitrary running mass scale”, Actually, further “no triviality” is also claimed.

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